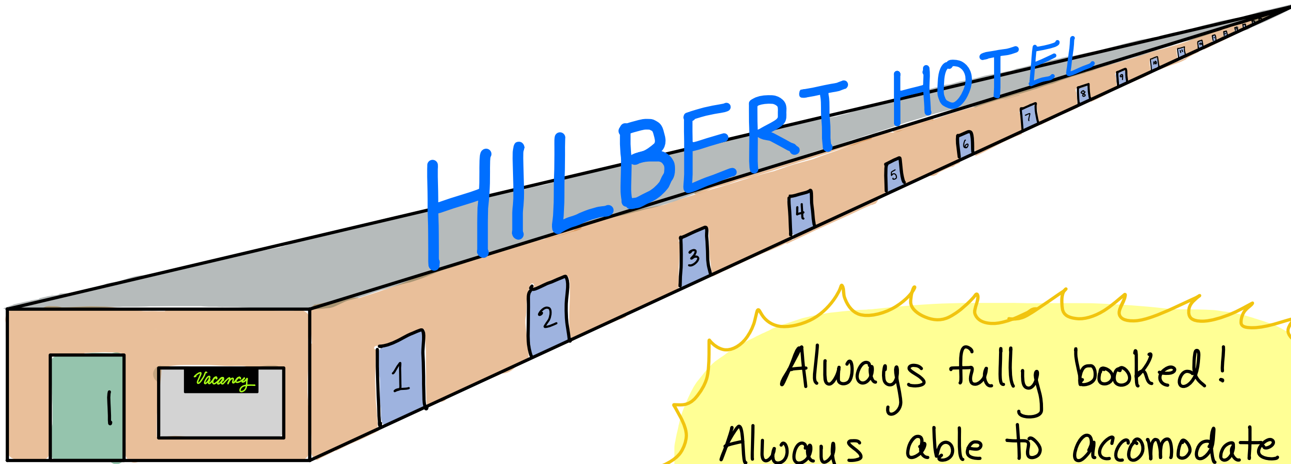
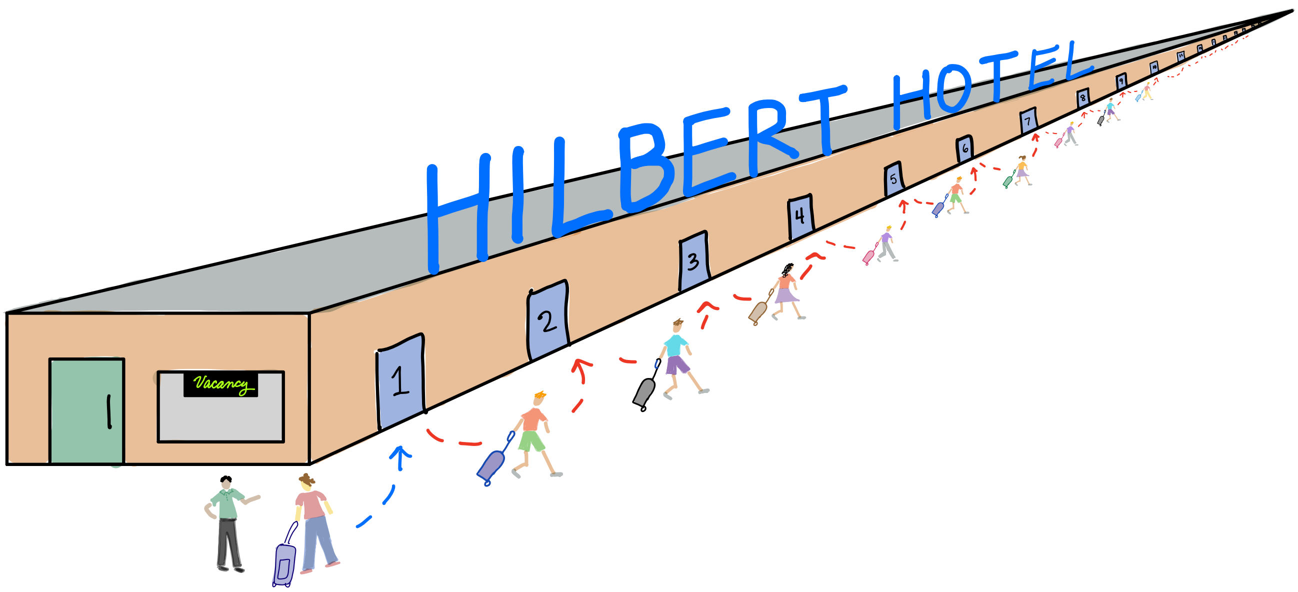
The Hilbert Hotel (Joy, Ch 30)

Cantor, set theory, the foundations of modern math

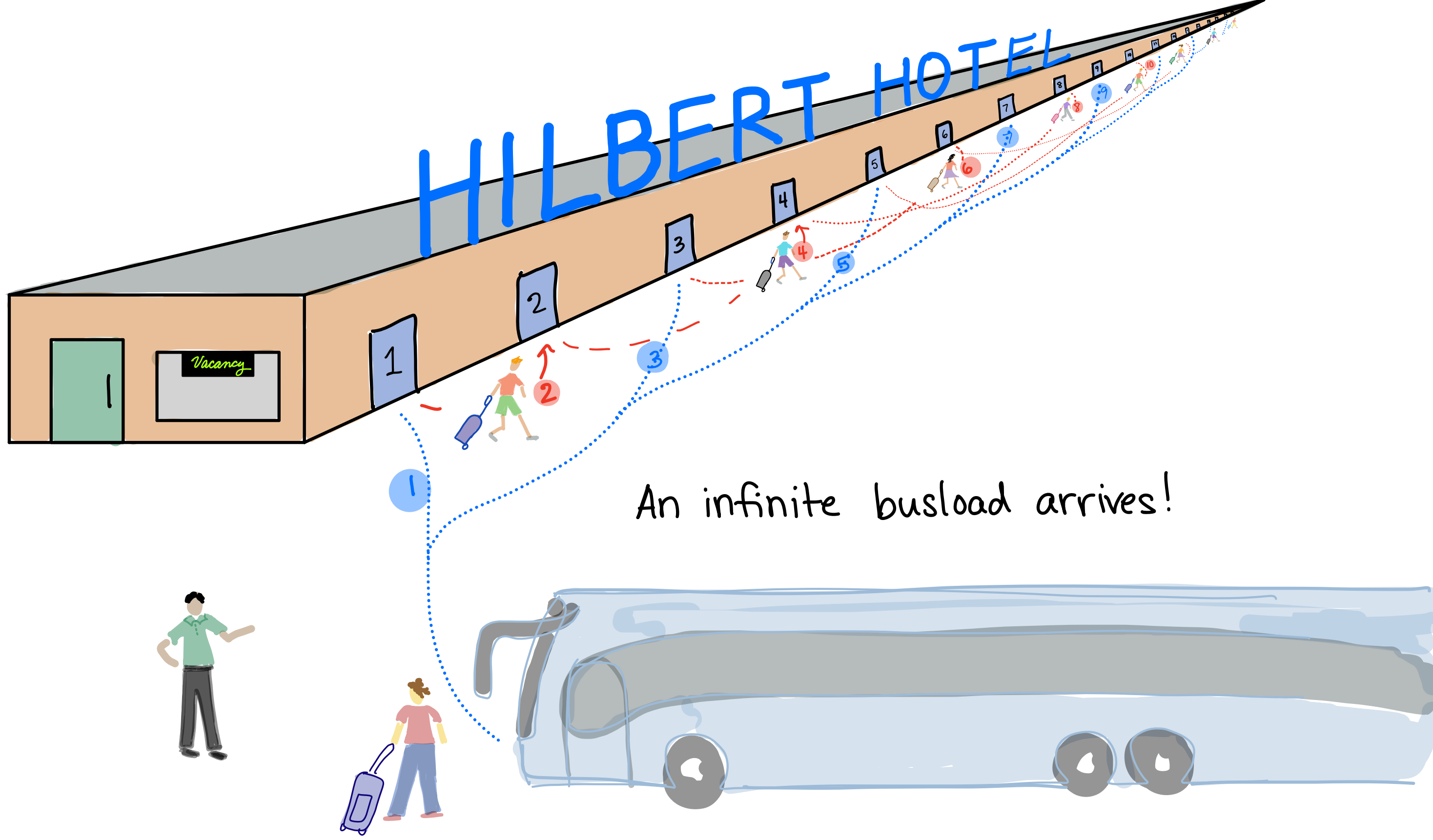
- sizes of infinities
- very controversial at the time



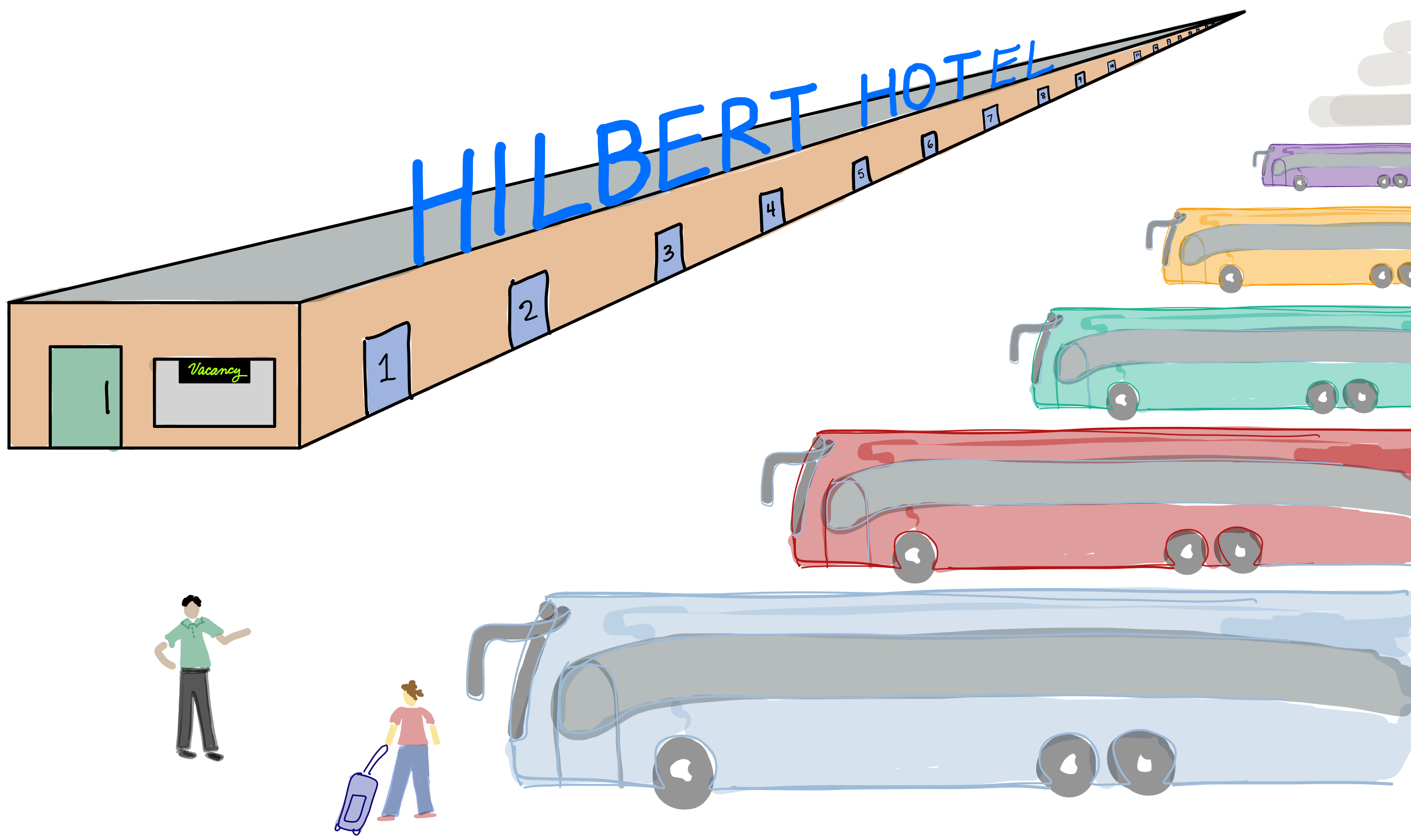
Always fully booked!
Always able to accomodate many additional guests!



New guest arrives — no problem: move everyone ______ over.



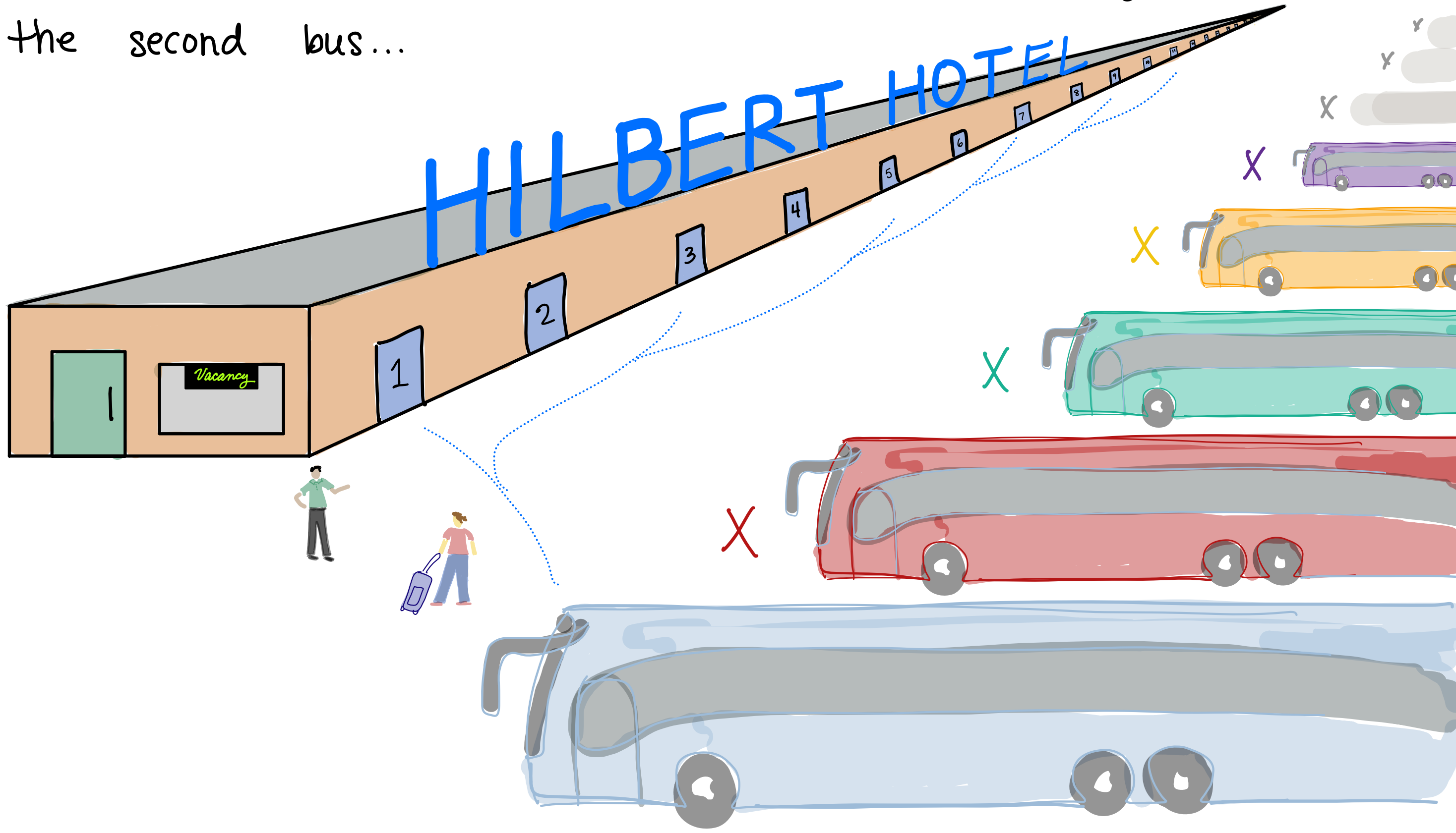
No problem: move current occupants to ______________ rooms.
Now ______________ rooms are _____ for new guests.



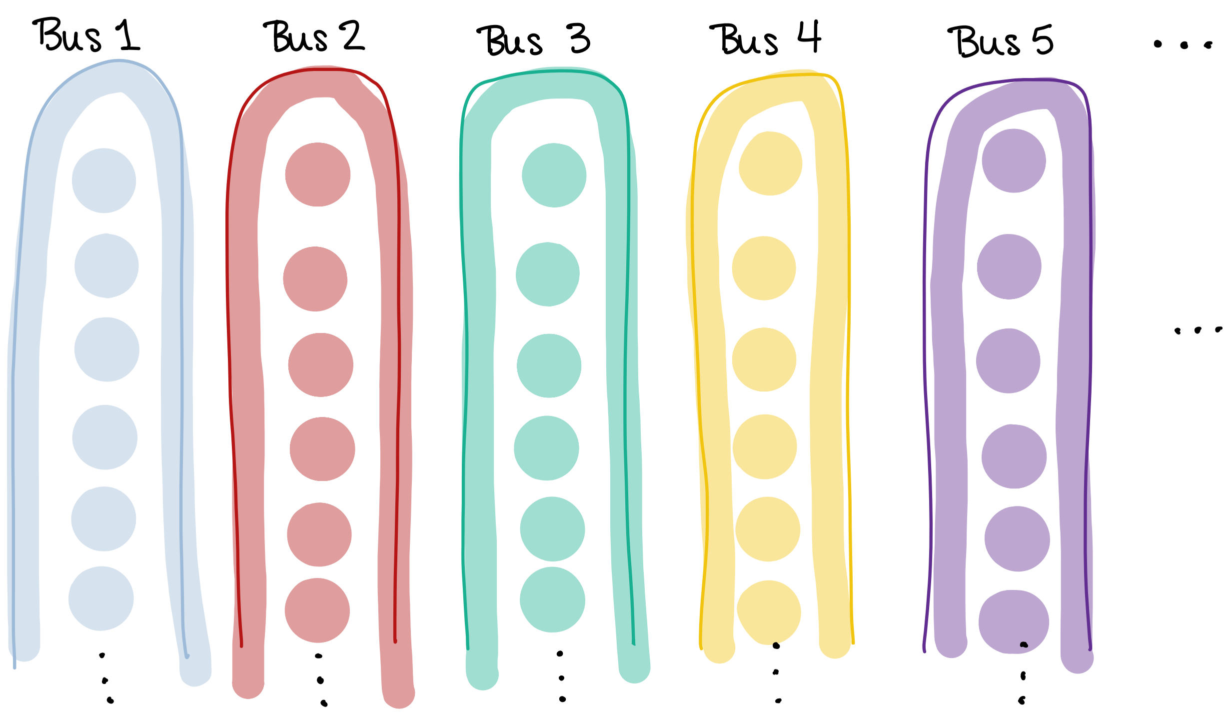
An infinite number of infinite busloads arrive...

Can free up all odd-numbered rooms as before, but how to accomodate new guests so that no one has to wait forever?

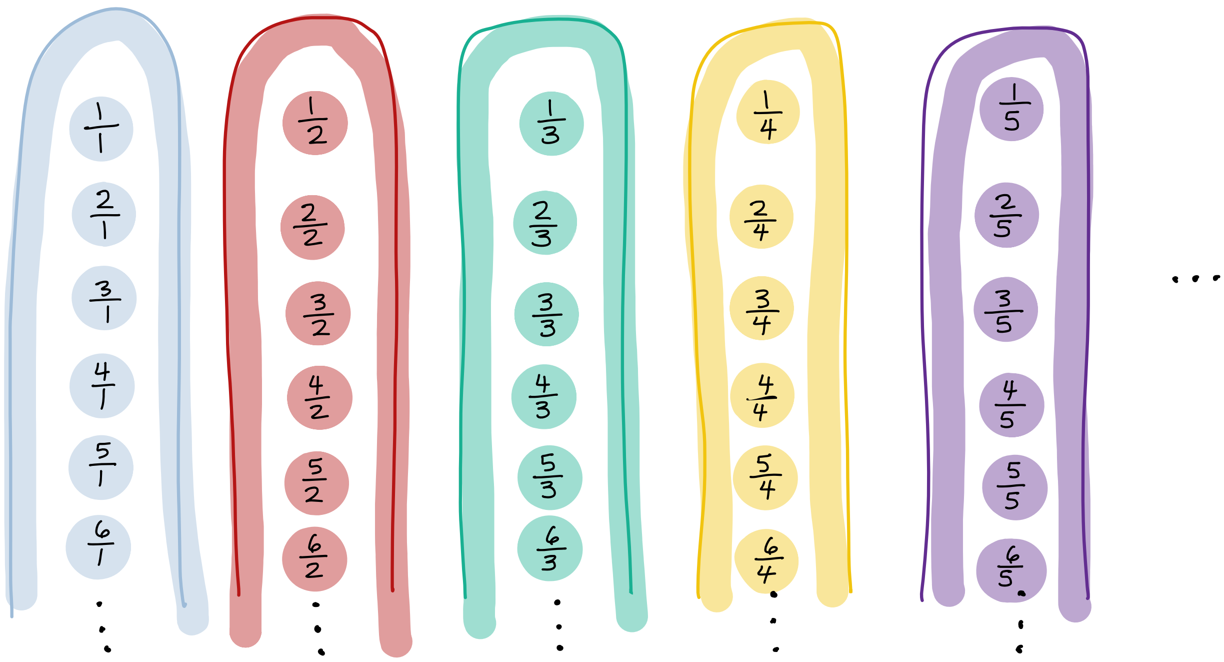
- If we wait to start unloading the second bus until after the first bus is unloaded... ______ will ______ get off the second bus...



Instead, accomodate them in this orderly way:

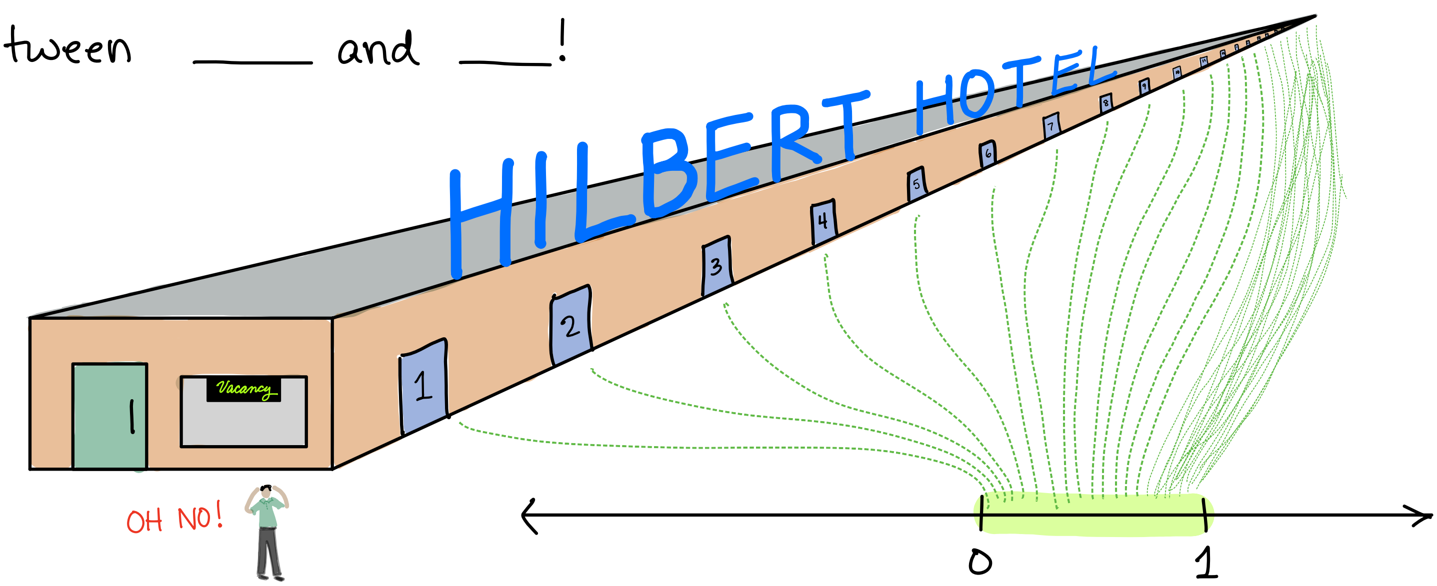


This is the essence of Cantor's stunning proof that the ____________ (bus passengers) are in 1-to-1 correspondence with ____________ 1, 2, 3, 4, ... (room numbers)!



The set of rational numbers & the set of natural numbers are the same "size" (more precisely, "________") Both are "____________"!

The set of ____________, however, is larger. It is ________ to accomodate all real numbers in the Hilbert Hotel. In fact, cannot even accomodate all real numbers between _____ and _____!



To prove this we will show that any way of putting these numbers into rooms will leave at least one number out.

Suppose we have put real #s b/w 0 & 1 into rooms:

Room 1	0.3781249...
Room 2	0.2267811...
Room 3	0.11311311...
Room 4	0.73567342...
Room 5	0.55689513...
⋮	⋮

We will show that there is a real number b/w 0 & 1 that has been left out.

		Build a number:	
Room 1	0.3781249...	0._ ...	(not 3)
Room 2	0.2267811...	0.__ ...	(not 2)
Room 3	0.11311311...	0.___ ...	(not 3)
Room 4	0.73567342...	0.____ ...	(not 6)
Room 5	0.55689513...	0._____ ...	(not 9)
⋮	⋮	⋮	

The number built in this way is between 0 & 1, but it is ____ any of the numbers in the rooms.

- Different from occupant of Room 1 (____ decimal place different)
- " " " " " 2 (____ " " ")
- " " " " " 3 (____ " " ")
- And so on.

Therefore, no matter how you arrange the rooms there will be at least one real number left out.

↪ The set of real numbers is "__________."